Parsing Patterns: Developing Metrics to Characterize Spatial Problem Solving Strategies within an Environmental Science Simulation

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Abstract: Urban planning and environmental science practice involves understanding and solving problems that arise from the intersections of human and natural systems. These systems often intersect with one another in very spatial ways: as when a roadway intersects a stream, or a cityscape covers a groundwater reservoir. When considering the challenge of integrating green infrastructure elements into urban landscapes (like water-trapping swales or areas of porous concrete) *where* these elements are placed, in relation to each other and in relation to other elements, can greatly modify their beneficial environmental impact. There is a growing need to help sensitize learners to such spatial patterns, especially as ordinary citizens are often called on to help make consequential urban planning decisions. Our research project has developed an interface (*EcoCollage*) that allows novice learners (undergraduate and AP high school students) to recognize spatial patterns present in environmental science / urban planning and mindfully rearrange them to affect emergent outcomes. To assist with this research, we developed software that applies spatial analytic techniques to the patterns co-constructed by learners, which allows us to detect and characterize their spatial problem-solving strategies. This paper describes how we conduct this spatial analysis and illustrates how it can be applied.

1. Overview

Spatio-temporal methods for studying learning are particularly useful when the content being learned is *itself* a phenomenon that unfolds over space and time, as is true with topics like environmental science and urban planning. Many of the current challenges we face as a society occur when human systems (e.g., settlements, roads, infrastructure) interact with natural systems (e.g., groundwater, habitats). The extent and nature of these interactions are dependent on the spatial patterns found within the human systems, the patterns within natural systems, and the intersections of these patterns. For example, the placement of wells is influenced by human patterns of settlement (wells need to be near where humans live) and groundwater reserves (wells need to be placed where groundwater is available), but the presence of other wells also affects placement (wells can create a "cone of depression" where the availability of groundwater is reduced). Urban planning and environmental science practice involves understanding and solving problems that arise from these human-natural intersections, but current educational practices delay opportunities to grapple with such problems until well into graduate-level studies. This can cause complications: a two-year graduate program may not be sufficient time for urban planning students to practice thinking spatially, and ordinary citizens are increasingly being involved in urban planning decision-making as stakeholders (Becu, Neef, Schreinemachers, & Sangkapitu, 2007) and may not be prepared to engage in the necessary spatial problem-solving. Thus, we have endeavored to create an educational intervention that allows spatial problem solving to be incorporated earlier in educational curricula.

Our research project has taken a design-based research approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) to develop an interface that allows novice learners (undergraduate and AP high school students) to recognize spatial patterns and mindfully rearrange them to affect emergent outcomes in human-natural systems (Shelley, Lyons, Minor, & Zellner, 2011; Shelley, Lyons, Shi, Minor, & Zellner, 2010). Novices, unlike practitioners or advanced students, often lack the specialized vocabulary needed to describe and discuss spatial patterns, making it difficult to assess their learning or study how their learning is progressing, as it is possible for learners to acquire spatial concepts weeks before mastering the disciplinary terminology (Singer, Radinsky, & Goldman, 2008). When conducting design-based research to build software to support novice learners in this process, then, it is critical to develop methods that can reveal learners' progress in early phases of learning. By observing student interaction and problem-solving as they used our interface, *EcoCollage*, we witnessed learners developing ad-hoc spatial problem-solving strategies. However, we soon realized that there were not very many viable approaches for documenting these strategies and their evolution.

A lack of mastery of disciplinary terminology can pose challenges for researchers looking for evidence of spatial reasoning, as methodological techniques for examining learning through dialogue (Bloome & Clark, 2006) require that researchers be able to identify the referents in learner's conversation. We found that even when applying a grounded theory approach (Strauss & Corbin, 1997), which makes no presuppositions about the nature of the content of learners' conversations, we had a difficult time establishing coding discriminations finer than "large-scale spatial" versus "small-scale spatial" (Slattery, et al., 2012). Arguably, incorporating gesture into the analysis would support a more refined coding scheme for spatial referents, but at the cost of increasing analysis time and thus the lag between design iterations.

A method for detecting spatial reasoning via an embedded assessment approach (Wilson & Sloane, 2000) would be a valuable tool for both researchers and educators. Owing to this same lack of expressive fluency, novice learners may know more about the system than they are able to express via standard summative assessment modalities like written tests. If the embedded assessment can be automated, additional learning supports like ongoing diagnosis, an oft-ignored feature of dynamic scaffolding (Puntambekar & Hubscher, 2005), become possible. Fortunately, the field of ecology has a long history of developing quantitative spatial analytic techniques (Dale, 2004), techniques which can be automated, and techniques which we can borrow to characterize the spatial properties of learners' intermediate solutions to spatial problems.

To assist with this research, we developed software that applies a particular spatial analytic technique, a variance-stabilized version of the Ripley's K spatial statistic, called the L function (Dale, 2004), to the spatial patterns co-constructed by learners. This statistic allows us to characterize a property of the intermediate spatial solution of learners that is particularly relevant to our problem space: the degree of relative dispersion of elements within a two-dimensional map of a region of land. This paper describes how this spatial dispersion statistic can be combined with other quantitative spatial descriptors to detect and characterizechanges in the spatial problem-solving strategies of learners.

2. Learning Scenario

When conducting urban planning, especially "green" urban planning, one must be aware of and sensitive to the spatial arrangements of elements belonging to both human and natural systems. Examples of natural elements might include specific animal habitats, the levels of elevation of the ground, or the distribution of soil types within a region. Examples of human elements might include roads, sewer systems, or the location of homes and other buildings. The placement and arrangements of these elements can have consequences for outcomes like flooding and groundwater infiltration. Sensitivity to such placements, and their emergent effects, is in keeping with the incorporation of systems thinking into recent redesign of the College Board's Standards for College Success (College Board, 2009), which are used to shape Advanced Placement tests and courses like AP Environmental Science.

Spatial Challenges in Green Infrastructure Planning

Green infrastructure planning requires that the planner be especially sensitive to the relative locations of both human and natural system elements. Green infrastructure is defined as "an interconnected network of green spaces that conserves natural ecosystem values and functions and provides associated benefits to human populations" (Schilling & Logan, 2008). Green infrastructure often takes the form of vegetated swales, green roofs and cluster development that aim to minimize urban stormwater runoff and associated pollution by using and mimicking natural systems to collect, treat, and infiltrate rain where it falls (Schilling & Logan, 2008). This is an important issue because changes in land cover are dramatic, widespread, and rapidly increasing across the globe, resulting in environmental consequences like habitat alteration and changes in hydrology. Currently, over 5% of the surface of the United States is covered by urban and other built-up areas (Montalto, et al., 2007), and urban areas are projected to increase to 9.2% of total land surface in the next 25 years (Alig, Kline, & Lichtenstein, 2004).

Creating swales is not cost-free, especially if land must be reclaimed from existing human-built structures like roads or parking lots. The trick to integrating swales into existing urban or exurban landscapes is thus to determine which locations will yield the highest benefits for both human systems (e.g., in terms of swale cost and reduction of problematic floodwater), and, ideally, for natural systems as well (e.g., in terms of the amount of rainwater that can be captured and converted to groundwater). Often, this is a tradeoff. Grappling with this tradeoff is a "wicked problem" in the sense that there will never be one "correct" answer (Rittel & Webber, 1973). Rather, learners are tasked with exploring a space of spatial arrangements of swales to discover a compromise between cost, flooding, and groundwater infiltration (Slattery, et al., 2012).

What makes this task difficult is that the spatial scale and pattern of swale placement which may be ideal for improving one starting scenario may not be ideal for the other scenarios, owing to the differences in the spatial patterns of other elements. The reason why exposing learners to this spatial problem space is important is that many existing urban planning "best practices" don't consider spatial interdependence and the resultant emergent outcomes. This can result in planning decisions that can have unintended outcomes or are so tokenistic that they may not even address the problem at all (as when the assumption is made that a green roof or two will compensate for acres of impermeable concrete). If learners are never asked to make explicit decisions about spatial placements of green infrastructure elements like swales, they will not have the opportunity to observe how different spatial patterns may drastically affect outcomes. Learners may come away from an environmental science course without any real understanding of how to productively grapple with real-world tradeoffs.

Figure 1. Three participants collaborate to place green infrastructure swales on a map of an urban landscape using the tangible *EcoCollage* interface. The map is a large sheet of paper, and the swales are tiles bearing printed symbols that can be recognized by a computer vision system viewing the map from a ceiling-mounted web camera. The locations of the tiles are fed into an urban storm water management simulation.

EcoCollage: a Tangible User Interface for Spatial Environmental Problem-Solving

We have augmented an urban storm water management simulation developed for the Illinois EPA with a computer vision input system to support a paper-based Tangible User Interface (TUI). TUIs may confer special advantages for spatial problem-solving (Antle, Droumeva, & Ha, 2009), especially for collaborative settings, and our paper-based approach is also designed to be cost- and time-effective for schools, as it allows students to interact with complex system simulations without requiring computer programming expertise or multiple computers. Rather, students solve environmental science problems through the hands-on placement of paper tiles (representing swales) on a large paper map (see Figure 1), which mimics authentic planning practice. The paper map is then read by a computer vision system and interpreted as input for the simulation (see Figure 2) so that students can test how each pattern affects the urban ecosystem. This vision system currently uses an inexpensive web camera mounted on the ceiling above the map, but a digital photo of the map would work just as well for supplying input to the simulation.

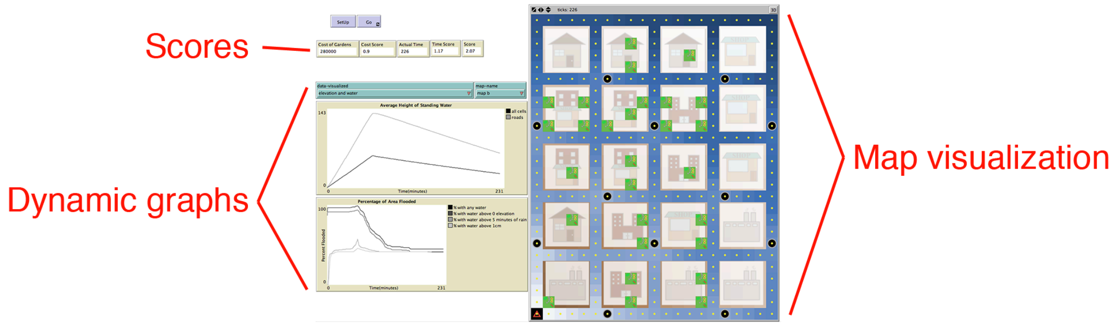


Figure 2. Annotated screen shot of the urban storm water management simulation. It is adapted from a simulation developed for the Illinois EPA, using the NetLogo agent-based modeling software. The simulation depicts the effect of a 100-year rainstorm on a 20-block urban area, in terms of the amount of rainwater that either floods the landscape, infiltrates into the groundwater supply, or gets drained away by the storm sewer system. The aim for learners is to reduce flooding while increasing infiltration, and simultaneously keeping costs (associated with installing new swales) low.

Learning Task

Learners are asked to place swales (indicated by green squares in the simulation) on a map of an urban landscape (see Figure 2). The challenge given to them was to balance human concerns (the cost of the swales and the amount of flooding present in the map) against natural concerns (the amount of groundwater infiltration). Each swale cost a fixed amount, but the amount of flooding reduction and infiltration increase that each swale could accomplish was highly dependent on the swale's placement. For example, although installing extra swales incurs additional cost (an undesirable outcome), users could increase the amount of infiltration (a desirable outcome) by clustering a number of swales together (essentially maximizing the return on swale cost). In another example, moving a swale from a location in the middle of a block to a location next to a sewer would result in a small decrease in infiltration (which is undesirable), but the flooding would decrease (a desirable outcome), as the swale would serve to trap surface water near the sewer, allowing a larger volume of water to drain into the sewer (sewer drainage occurs at a faster rate than infiltration). The task thus required that learners be sensitive to both univariate patterns (the location of swales *vis-a-vis* other swales) as well as to multivariate patterns (the location of swales *vis-a-vis* man-made infrastructure elements like impermeable road surfaces and sewers). Ideally learners would explore a number of these patterns, experimenting with placing swales close or far from one another and close or far from other map features.

3. Spatial Research Challenge

Assessing the progress learners made in apprehending and responding to spatial patterns requires that we first be able to characterize these patterns ourselves. We found that the ontologies of "spatial knowledge" that have been developed to describe how students learn geospatial concepts (e.g., Marsh, Golledge, & Battersby, 2007) were inadequate for characterizing the sophistication of such patterns, rooted as they are in traditions like geospatial mapping, and thus tending to be more concerned with defining paths and regions than distributions of elements across space. Moreover, because there is no one "right answer" to green infrastructure problems, the problem solving process is often more important for learning than arriving at the "correct" solution. So, then, we would like to ensure that learners experiment with a variety of characteristically different solutions. Traditional assessment methods might look at measures like time-on-task, transcriptions of the discussion, or scores on pre/post tests, and indeed these measures are of interest to us, but these measures say nothing about the exploration path taken by the learners. In our context, the exploration path consists of a series of spatial arrangements of swales on a map with pre-existing patterns of human and natural elements. This requires that learners exercise a specific component skill of visual reasoning where they attend to the relations between objects. This component skill of detecting object-object relations seems to be distinct from other classic forms of spatial reasoning skills, like dynamic transformations of objects (Hegarty, 2010), and from other types of visual reasoning like object visualization (Kozhevnikov, Kosslyn, & Shephard, 2005).

To illustrate why we need to understand the exploration path of users, imagine a group that decides a priori that placing swales near low-lying areas is the best idea. They may iterate on this basic strategy, adding or removing a few swales here or there, but never deviate markedly from the basic idea. This approach to problem space exploration is known as "hill-climbing" strategy in artificial intelligence. In contrast, other learners might engage in what is known as a "random walk" strategy – trying out a series of wildly different spatial arrangements – an approach similar to when users slam slider bars from one end to another when exploring traditional simulations. But we know from research that slider-slammers, or "oscillators," don't necessarily build a good sense of how the simulated system works (Levy & Wilensky, 2005), since the extreme configurations tend to have so little in common in terms of outcomes. We also know from artificial intelligence that neither hill-climbing nor random-walk searches are guaranteed to converge on a satisfactory outcome, and further, we have good reasons to believe that neither strategy alone is effective for learning about the underlying system. Thus, things we might wish to track in learners include the *breadth* of their explorations, in other words, the sheer variety of different spatial patterns they explored, as well as the *depth* of their explorations, i.e., the number of "variations on a theme" of spatial arrangements that learners have explored. Being able to detect the exploration behaviors in learners is a first step towards confirming this suspicion that some kinds of exploration strategies may be more productive for (1) learning about spatially-mediated interactions between human and natural systems, and (2) for acquiring experience with negotiating compromise solutions, which is a disposition valuable for green infrastructure planning. Once that is known, we will also have the tools to be able to detect when learners seem to be on an exploration path unlikely to maximize learning, so we can then intervene and urge learners to consider a different exploration strategy.

4. Key Methodological Decisions in Approaching the Spatial Research Challenge

In order to track learners' trajectory of spatial pattern exploration, we need some way of characterizing these spatial patterns. Fortunately, we have some tools that we can borrow from ecology: a number of ecologists make use of spatial statistics to better understand the spatial distribution of plants and animals within an ecosystem. In addition to more standard measures like abundance, there are an array of spatial analytic techniques for detecting spatial inter-relationships that can be used to characterize swale placement strategies (Dale, 2004). Some statistics can describe characteristics about the placement of organisms *vis a vis* each other, as when cacti grow in clumps of a certain proximity, or *vis a vis* other spatial elements, as when certain kinds of flowers tend to grow near certain kinds of trees. The next section explains how one such measure is calculated;

Ripley's L Estimate of Spatial Dispersion

One common measure is known as *Ripley's* K (Dixon, 2006), which, put crudely, indicates whether items are clumpy or spread out (i.e., "overdispersed"), relative to one another. The essential concept is a simple one: for a given radius, *r*, swept out from elements of interest (e.g., sewers), how unusual (given the overall density of elements on the map) is it to encounter elements of interest within an area of that size (either the same category, or another category of element entirely, e.g., swales):

$$K(r) = \forall i \in C_1, j \in C_2 \frac{1}{\lambda_1 \lambda_2 A} \sum_i \sum_{j, j \neq i} w_{ij} I(d_{ij} < r)$$

Where:

- A is the area of the map
- λ_1 is the average density of points of that category C_1 throughout the map (i.e., the total number of points in C_1 divided by the total area of the map), and λ_2 is the average density of points of that category C_2
- We iterate across elements *i* that belong to a primary category of interest C_1 (which could be swales, or sewers)
- For each *i*, we iterate across all elements, *j*, that belong to a secondary category of interest C_2 (which in our case would be swales), where *i* and *j* are not the same element
- w_{ij} is the proportion of the area of the circle of radius d_{ij} , the distance between centered at that exists inside the map (see Figure 3)
- The indicator function *I* returns 1 if the distance between *i* and *j* (d_{ij}) is less than the radius *r*, 0 otherwise

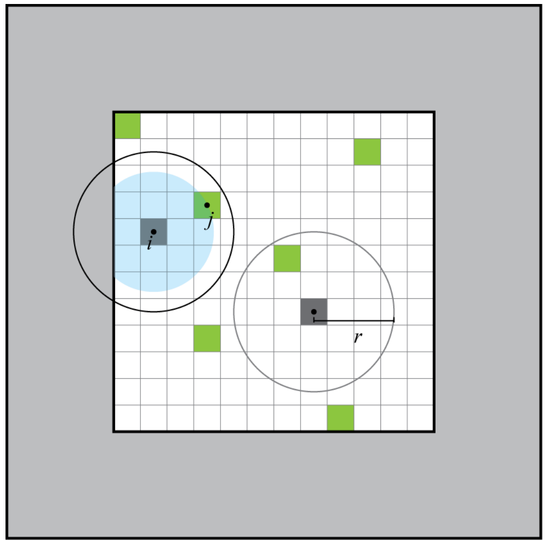


Figure 3. Illustration of how one estimates *Ripley's K* for a given spatial distance (*r*). The map is divided into a grid and populated with two categories of elements (the grey and green squares). Iterating through one category (grey squares) one simply counts how many of the second category (green) are within distance *r*. The blue shading indicates the weighting (w_{ij}) one must perform.

So with the *Ripley's K* equation, we would expect that if the points of category C_2 were randomly distributed relative to category C_1, we would expect the *K* value for a given spatial distance *r* to be equal to the area of a circle of size *r*. To allow us to more easily make judgments about whether a *K* value indicates overdispersion or clumping, we often make use of the *Ripley's L* function, which transforms the *K* function relative to the radial area:

$$L(r) = \sqrt{\left(\frac{K(r)}{\pi}\right)}$$

With the Ripley's L formulation, we know that if the C_2 points are randomly distributed relative to category C_1 , $L(r)$ should equal *r*. Values of $L(r) - r > 0$ indicate that we are seeing more points of type C_2 within radius *r* than we would expect, given chance, indicating that C_2 is clumped relative to C_1 at the spatial distance of *r*. Values of $L(r) - r < 0$ indicate that we are seeing fewer points of type C_2 within radius *r* than we would expect, given chance, indicating that C_2 is overdispersed relative to C_1 at the spatial distance of *r*.

Application of Ripley's L Estimate to Our Problem Space

With a spatial analytic approach, we are better able to characterize certain aspects of the spatial strategies being employed by users, and thus are able to track their progress in exploring the possible solution space. Specifically, the spatial dimensions we can track are along two axes:

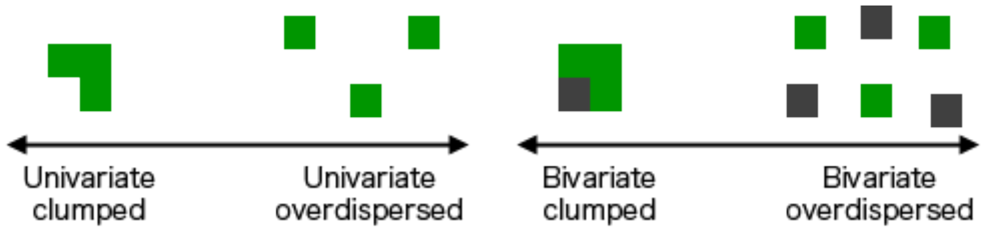


Figure 4. Illustration of the spatial properties that univariate and bivariate *Ripley's L* estimates allow us to track.

The following discussion presents some examples of how the Ripley's L estimate may be used to characterize the univariate and bivariate distributions present in learner's map arrangements.

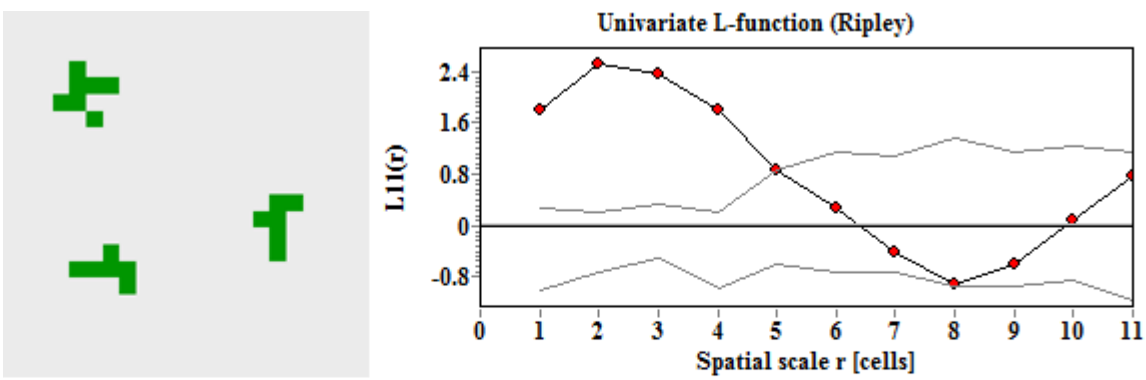


Figure 5. On the left is a 2D spatial pattern of items (e.g., swales, represented as green squares) where the individual items are clumped together in small groups. The right shows the Ripley's L estimate (*L(r) – r*) for distances of 1 to 11.

The univariate *Ripley's L*-function analysis shown in Figure 5 indicates that for distances from 1 to 4, the swales are significantly clumped together (the grey lines on the *L* function graph indicate 99% confidence intervals). This should be expected, looking at the map, as each swale is within a radius of 4 of every other swale. By contrast, Figure 6 shows the univariate *Ripley's L*-function analysis of a map where, at certain spatial scales (distances of 2-5, and again at 8), the swales are significantly overdispersed relative to one another. The significant overdispersion at the distance of 8 owes to the regularity of the pattern: if each item is roughly 4 squares away from the others, we would expect to see overdispersion at radii that are multiples of 4: 8, 12, and so on.

It should be noted that when items are overdispersed, they are not placed randomly (which would allow them to be arbitrarily close to one another), although people often confuse the two ideas. (When one asks a person to place items "randomly" they tend to create an overdispersed distribution, perhaps indicating an underlying misconception of what a random distribution is). Rather, the arrangement is more akin to the placements items take when they repel one another (e.g., magnets of opposing polarities).

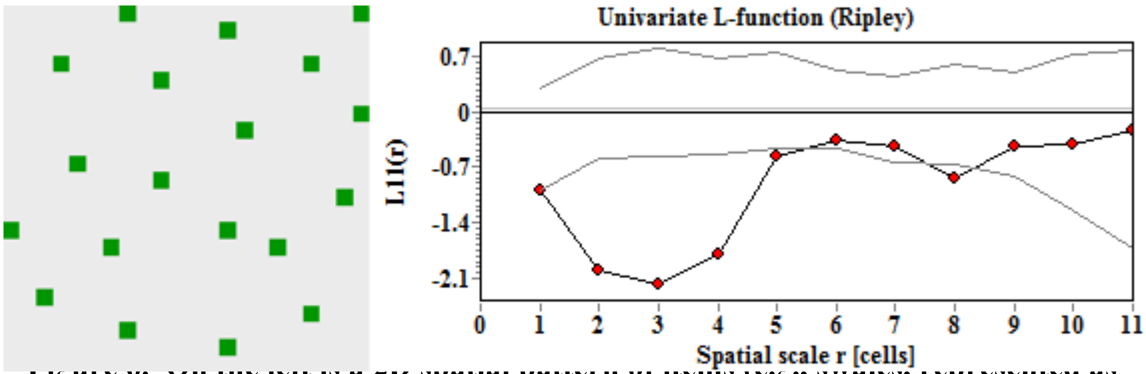


Figure 6. On the left is a 2D spatial pattern of items (e.g., swales, represented as green squares) where the individual items are overdispersed. The right shows the *Ripley's L* estimate for distances of 1 to 11.

Thus far we have illustrated how *Ripley's L* can be used to estimate univariate clumping/dispersion, as shown in Figure 5 and Figure 6, which indicates how items are placed relative to other items of the same kind. *Ripley's L* can also be used to estimate *bivariate* clumping/dispersion, which occurs when you place items of one type near or far from items of a second type. Figure 7 shows a map that contains both swales (green) and sewers (dark grey). The univariate *Ripley's L* estimate (top graph) shows that swales are clumped with each other at distances of 1-2. The bottom graph shows the result of the bivariate *Ripley's L* estimate: how clumped/dispersed the swales (green) are with respect to the sewers (grey). One should note that the swales are significantly clumped *vis a vis* the sewers at a scale of 1-2, which we should expect from their close adjacency.

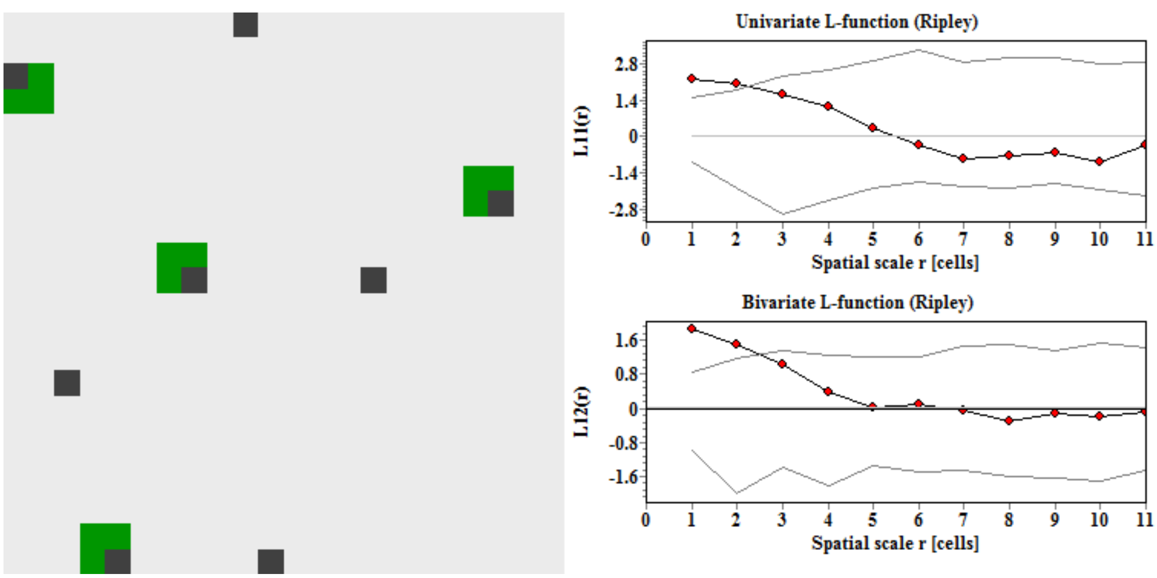


Figure 7. On the left is a spatial pattern of two different categories of items: swales (green) and sewers (grey) The two graphs at right present the univariate *Ripley's L* estimate (top) of swale-to-swale distribution, and the bivariate *Ripley's L* estimate of how swales are distributed relative to sewers.

The use of both univariate and bivariate *Ripley's L* estimates allows us to disambiguate spatial patterns like that shown in Figure 7 from patterns like that shown in

Figure 8. The univariate *L*-function is very similar to that shown in Figure 7, as the swales in both maps are significantly clumped at distances of 1-2. The bivariate *Ripley's L* estimate shows us that for radii of 2-4 around sewers, however, swales are much less likely to be found than one would expect by chance. Moreover, once the radius is extended to a size of 7-8 around the sewers, one once again finds more swales than the overall density of swales might predict. Being able to detect differences between maps like those in Figure 7 and Figure 8 is meaningful in our spatial problem space, as proximity and distribution can play a role in how different elements interact to produce outcomes. For example – placing swales near sewers as in Figure 7 will decrease the amount of flooding experienced during a rainstorm at the expense of groundwater infiltration, while placing them in an overdispersed fashion *vis a vis* the sewers as in Figure 8 will increase infiltration at the expense of increased flooding.

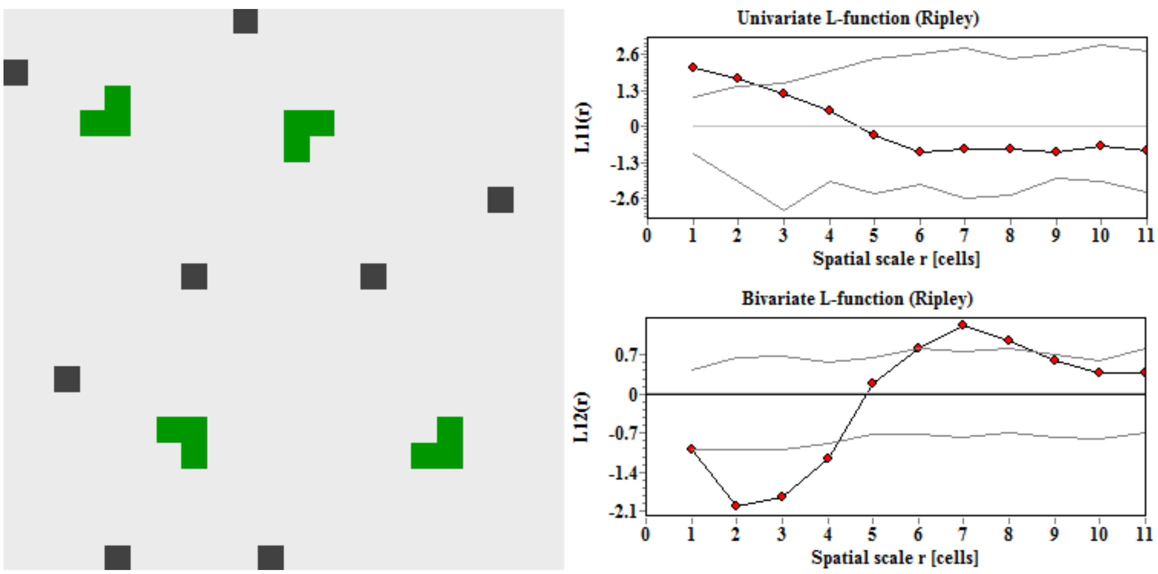


Figure 8. This map is an example of how both clumping and overdispersion can be present in the same arrangement. The two graphs at right present the univariate *Ripley's L* estimate (top) of swale-to-swale distribution, and the bivariate *Ripley's L* estimate of how swales are distributed relative to sewers

An attentive reader will have noticed that in the examples in Figures 5-8 we have calculated the *Ripley's L* estimate for a number of radii – from 1 to 11 (typically, the largest radii one analyzed using *Ripley's L* estimates is equal to half the size of the map, which in this case is 22). Unless we are interested in detecting a very specific distance between items, a univariate pattern cannot be described with a single *L* value – we must use an array of *L* values across a range of scales ($r_1, r_2, r_3, \ldots r_n$), where n is the largest distance we might be interested in. So each map would be described with a *tuple* of n items. The univariate distribution of swales on the map presented in Figure 5 would be described by the following 11-tuple:

$$L_U = [1.8, 2.4, 2.2, 1.8, 0, 0, 0, 0, 0, 0, 0]$$

Notice that the last 7 values are set to 0 – because these *L*-values lie inside the 99% confidence intervals, they are statistically indistinguishable from a random dispersion. In other words, it cannot be said with confidence that the swales are either clumped or overdispersed at radii 5 – 11. The tuple can be simplified further because we don't care overly much about the *exact* *L*-value estimate – just whether it is *significantly* clumped or significantly overdispersed, so we can replace positive values in the tuple (indicating clumping) with 1s, and negative values (indicating overdispersion) with -1s; here we show the substitution for Figure 5's L_U:

$$L_U = [1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0]$$

For maps with multiple categories of interest, we will also need to compute bivariate *Ripley's L* values, again, for all *n* radii of interest. So, to describe how swales are placed *vis a vis* sewers, we need to have an array of bivariate L values. Here we show the bivariate *Ripley's L* tuple, L_B, for in the map presented in Figure 8:

$$L_B = [0, -1, -1, -1, 0, 0, 1, 1, 0, 0, 0]$$

If we are interested in both swale-swale placements and swale-sewer placements, a full spatial distribution description would require that we specify *both* the univariate and bivariate *Ripley's L* estimates. L_U and L_B. For example, here is the full description for the map in Figure 8:

$$L_U = [1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$L_B = [0, -1, -1, -1, 0, 0, 1, 1, 0, 0, 0]$$

Once we have these descriptions, we can easily compare maps to see if they differ in terms of their univariate or bivariate spatial distributions. This can be useful both for comparing the spatial properties of different solutions, as well as performing step-by-step comparisons to determine if with each change to their maps learners are altering their strategies with respect to univariate or bivariate spatial placements. To compute the Spatial Dispersion Dissimilarity (SDD) across two maps, *a* and *b*, we need only to compute a variant of the Hamming distance across the *tuples*, where we normalize the value across the number of radii, *r*, used in the spatial dispersion computations:

$$SDD = \frac{1}{r}\sum_r \left|L_r^a - L_r^b\right|$$

So if we wanted to compare the map in Figure 8 to the map in Figure 7, the univariate *SDD*, SDD_U, is:

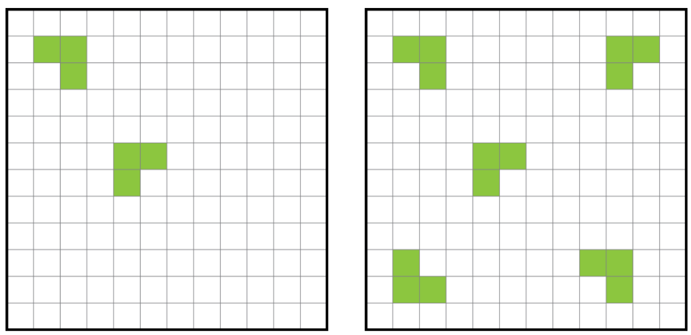
$SDD_U = (|1-1| + |1-1| + |0-0| + |0-0| + |0-0| + |0-0| + |0-0| + |0-0| + |0-0| + |0-0| + |0-0|) / 11$
$SDD_U = 0$

While the bivariate *SDD*, SDD_B, is:

$SDD_B = (|1 - 0| + |1 + 1| + |0 + 1| + |0 + 1| + |0 - 0| + |0 - 0| + |0 - 1| + |0 - 1| + |0 - 0| + |0 - 0| + |0 - 0|)/11$
$SDD_B = (1 + 2 + 1 + 1 + 0 + 0 + 1 + 1 + 0 + 0 + 0)/11 \approx 0.64$

Augmenting Ripley's L with Other Spatial Metrics

The above illustrates how we can condense the comparison of the spatial dispersion patterns of two maps into a simple pair of numbers, one for the univariate Hamming distance between the maps' *Ripley's L*-estimates, SDD_U, and one for the bivariate Hamming distance SDD_B. Unfortunately, we still need more descriptors to better characterize the spatial similarities/dissimilarities of two maps. We run the risk of mis- or under-characterizing learners' spatial reasoning strategies by only relying on *SDD*, as it speaks only to very specific kinds of spatial properties. To build a better characterization we need to add extra information, like the number of the swales used, as the Ripley's L measure alone could lead us to inappropriately conclude two patterns are very similar, as in the example in Figure 9.



L_U= [1,1,0,0,1,1,0,0,0,0,0] L_U= [1,1,0,0,1,1,0,0,0,0,0]

Figure 9. Two maps with identical univariate *Ripley's L* estimates but very different numbers of swales.

Thus we also need to measure the difference in abundance of swales, *AD*. For our purposes, we want *AD* to be *proportionally* sensitive to changes in the count of swales, so we will not normalize over the size of the map, but rather, over the maximum number of swales in either of the two maps being compared. (The difference between 5 and 15 swales should be "larger" than the difference between 35 and 45 swales). Assuming we have two maps *a* and *b*, each with *N* swales:

$$AD = \frac{|N^a - N^b|}{MAX(N^a, N^b)}$$

The incorporation of *AD* will help us disambiguate some spatial strategies that gare indistinguishable by using *Ripley's L* alone, but not all. Similarly, because *Ripley's L* estimates are sensitive to relative, not absolute distance, two strategies might have the same *L*-tuple but actually represent very different spatial strategies, as shown in Figure 10.

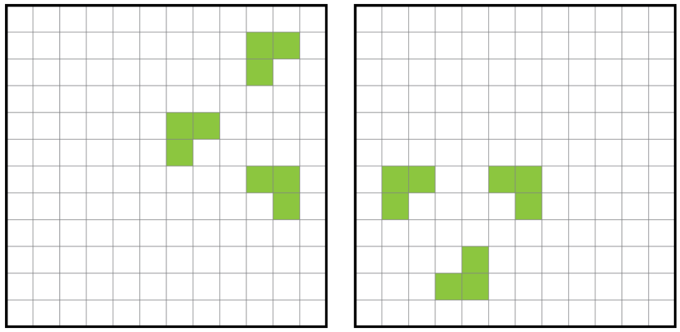


Figure 10. Two maps with similar univariate *Ripley's L* estimates and identical numbers of swales but different overall placements.

There are many computational approaches to characterizing the difference in the exact placements, *PD*, between the two maps in Figure 10. For simplicity, we have decided to use once again a variation on the Hamming distance computation, called *edit distance*, to distinguish between such strategies. Imagine that a map of size *h* horizontal locations by *v* vertical locations. It can be described by an ordered list of $h*v = n$ numbers, with each number representing the category of square present at each location. The edit distance is the sum of the squares in the first map's ordered list, M^a, whose categorical numbers do not match their corresponding square in the second map's ordered list, M^b. For Figure 10, the edit distance would be 18 (a way to think of it, visually, is to imagine lining the maps up on top of one another, and counting all the squares that are not the same on both maps). This can be normalized using the total number of swales used in each map, N^a and N^b, so for Figure 10, the Placement Dissimilarity would be 1, indicating that no swales are placed in the same locations. The equation for *PD*:

$$PD = \frac{1}{\left(N^a + N^b\right)} \sum_{1 \le i \le n} \left(M_i^a \ne M_i^b\right)$$

Examples of Analyses Made Possible with Spatial Metrics

For our purposes, these four kinds of measures, the univariate and bivariate Spatial Dispersion Dissimilarities (SDD_U, SDD_B), the Abundance Dissimilarity (*AD*), and the Placement Dissimilarity (*PD*) are sufficient for detecting meaningful shifts in strategy in our problem space. This quantitative characterization of strategy has research benefits, in that it allows one to study how learner's strategies change over time, and in turn to determine which strategic shifts or combinations of strategic shifts result in better solutions. There are pragmatic benefits as well. Because this method allows us to detect shifts in learner strategies automatically, in real-time, as learners interact with a spatial problem solving simulation, we can use that information to provide real-time guidance in a fashion not to dissimilar from cognitive tutoring software (e.g., Koedinger & Corbett, 2006). (Of course, as with cognitive tutors, a large database that characterizes productive versus unproductive solutions paths would first have to be compiled for each problem space, although some of this corpus, too, could be automatically generated).

Here we turn to an example taken from our initial pilot study (Shelley, et al., 2011; Slattery, et al., 2012), wherein we asked pairs of learners to try to generate an optimal solution to the green infrastructure rainwater management problem described above. A few caveats: in this pilot study, restrictions we had placed on where swales could be located meant that performing a univariate analysis of swale-swale placement would be uninformative, so we only used the SDD_B, *AD* and *PD* spatial metrics. We can now compare how one spatial map produced by learners compares to another along each of these four dimensions, to characterize what kinds of spatial strategies are dominating. For example, in Table 1 we show the SDD_B, *AD* and *PD* comparison metrics across the seven different spatial arrangements tested by a dyad of participants. (Note that we used a threshold function to convert each of the fractional SDD_B, *AD* and *PD* values into Boolean numbers, where 1 is indicative of a meaningfully large shift in that metric, and 0 is indicative of only a small change in that particular spatial strategy. Each metric has its own threshold function, as what constitutes a meaningfully large difference can be small for some metrics and large for others. Each threshold is determined by the nature of the problem space). One can see that after testing the initial map, Map 1, the dyad made meaningfully large changes in both the number (*AD*) and placement (*PD*) of swales. No large shifts in strategy occurred between Maps 2 and 3, indicating that the dyad merely refined their Map 2 strategy. In shifting from Map 3 to Map 4, however, we detect meaningful shifts in both placement of the swales (*PD*) and in bivariate distribution of the swales relative to the sewers (SDD_B). Towards the end of the episode, most of the strategic changes taking place concern the placement of the swales *vis a vis* the sewers.

Table 1. Illustration of how a dyad in our pilot study shifted in strategy during an exercise where they iteratively generated and tested 7 spatial arrangements of swales.

Map Comparison	SDD_B	*AD*	*PD*
1-2	0	1	1
2-3	0	0	0
3-4	1	0	1
4-5	0	0	1
5-6	1	0	0
6-7	1	0	0

We can study other aspects of the learners' problem solving as well. For example, if we wanted to determine if exploiting one strategy at the expense of the others (a hill-climbing approach) is effective in our problem space, we can compute the proportion of map changes within a session that were dominated only by that strategy, and correlate that number with the maximum score obtained during a session, across all sessions with all participants. This in fact shows us that hill-climbing alone is not sufficient: neither the presence of SDD_B hill-climbing, the presence of *AD* hill-climbing, nor *PD* hill-climbing correlates with scoring well (on either optimizing the cost of the green infrastructure or optimizing drainage of water). When one examines the presence of strategy shifts made in conjunction with one another (as occurs in the Map 1-2 and Map 3-4 shifts in Table 1) one does see significant correlation for some strategy combinations. In particular, when meaningful shifts in bivariate distribution are coincident with meaningful shifts in either *AD* ($r = 0.55$, $df = 30$, $p < 0.01$) or *PD* ($r = 0.42$, $df = 30$, $p < 0.02$), we see a significant positive correlation with the maximum cost-minimization score the dyad is able to obtain.

This indicates that when the learners leverage an awareness of swale-to-sewer distributions when altering the number or placement of swales, groups are able to get the most "bang out of the buck" for each swale they place. Conversely, there is no correlation between coincident *AD* and *PD* shifts and the cost-minimization score ($r = -0.08$, $df = 30$, $p = 0.66$), which shows that learners must become aware of the importance of the placement of swales in relation to sewers.

5. Advantages and Tradeoffs

Using spatial metrics allows us to detect nuances in learner's spatial reasoning that would otherwise be hard to disambiguate. For example, in prior attempts at using grounded coding to understand learners' spatial reasoning we found that learners seldom explicitly stated their spatial strategies (Slattery, et al., 2012), so it was difficult to understand the path of their exploration. This technique allows us to quantifiably characterize their strategies, and shifts in strategies, in spatial terms.

There are still limitations to this approach, however. For example, one might question the extent to which a change in the number of swales constitutes a "strategy shift" in its own right – when people conceive of strategies, they typically have a much richer way of characterizing their intentions. For example, many of the learners in our pilot study were observed to add more swales in response to dynamic outcomes from the prior map's arrangement. In some cases, the dyads added extra swales in locations that exhibited puddling, a sort of "sponge" strategy. In other cases, dyads were observed to add swales in locations in strategic positions near where concentrated "streams" of water formed, a sort of "water diversion" strategy. Using our quantitative metrics, we can detect when learners make these changes, but we lose the nuance motivating the changes.

The best way to think of this approach to quantitatively characterizing spatial problem solving is to view it as yet another tool that researchers can use to better understand learner intentions and actions. We plan not to use it as a stand-alone measure, but to combine it with the aforementioned dialogue analysis, and with a forthcoming gesture analysis we hope will reveal some of the ways learners are responding to the emergent dynamics of our problem space.

6. Comparisons to Other Papers in the Session

Paper 1: Making the Absent Present: Improvised Representational Fields in Students' Negotiations of Meaning with GIS

We share with Paper 1 an interest in studying how learners' reasoning about spatial phenomena change over time. Although we did not track the gestures in this work we, too, witnessed a number of gestures being introduced and imitated as groups negotiated future actions and interpreted the reaction of the simulation to their actions. In our case, the representational field is partially defined for the learners, in that we provided a large paper map with tangible tokens which the learners used as a "stage" for their often gesture-rich discussions.

We saw learners developing gestures for signaling the boundaries of areas of concern, such as where water pooled, and gestures that represented the dynamic properties of the simulation, like the paths and rate of water flow across the surface. The approach developed in Paper 1 may very well inform our next analysis. For example, it would be interesting to discover if learners who make gestures indicating that they are attending to the dynamic processes of the model explore characteristically different spatial configurations than those who do not make such gestures. We are interested in being able to detect dynamic thinking because while an understanding of dynamic processes is just as important a skill as predicting outcomes when it comes to solving environmental science problems, it is often under-stressed in environmental science curricula

Understanding of how dynamic processes unfold allows people to design and enact better amelioration strategies – by way of example, while predicting how much oil from an ocean spill will reach shore helps one judge *how many* oil-absorbing booms to use, understanding the currents that will bring the oil to shore allows one to decide *where* to place those booms to best effect. The unique value that complex system simulations can bring to environmental science education is that ability to witness and experiment with dynamic processes – and we are invested in developing methods to understand how learners perceive and respond to these dynamic processes so that we might stress this under-emphasized part of their education.

Paper 2: Constructing Quartets: A Framework for Analysis in Musical Groups

This paper developed techniques for detecting and studying the use of gestures used within a group performance to help negotiate and orchestrate individual performances. Our current setup does not demand "live", performative coordination: our groups are free to negotiate and re-negotiate a spatial configuration until they have reached a static arrangement that suits them. Therefore there is not a lot of overlap between Paper 2 and ours, although one can imagine that if we were to adopt a "real-time", as opposed to "turn-based", style of interaction with the simulation, we too would need to attend to how the learners coordinated the orchestration of their spatial solutions. This kind of setup might be useful for environmental science problems that in fact demand more of a dynamic response, as in the oil spill example mentioned above.

Paper 3: Map Performances: Expanding Spatial Thinking with Embodied Activity

This paper also appears to be integrally related to performance, but in this case, where a single actor is orchestrating the interaction with and presentation of a multi-media geographically-referenced information. The discoveries the authors make concerning the types of gestures developed by the learners to illustrate spatial ideas that will likely inform the gesture analysis we wish to perform in future work (cf. Paper 1).

Paper 5: Fostering Mathematical Discovery: One Tutor's Strategies for Ushering the Construction of Proportional Schemas Via Mediated Embodied Interaction

We share an overarching interest with Paper 5 in the "show and don't tell" pedagogical approach of guiding learners towards perceiving certain spatial patterns, and Paper 5's terming this guiding process "re-orientation" rather than "orientation" also suits our epistemological stance: we welcome and expect learners to approach the challenge from a number of possibly equally-valid perspectives, and in fact, we expect that any guidance we develop would lead learners through a series of re-orientations to different spatial patterns. Like Paper 5, our main entrée into the reasoning processes of our learners comes via an analysis of their actions, although in our case, we hope to offload the hard work of tracking student progress to the software through an automated detection of spatial patterns.

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